Mechanical Engineering – Scientific Journal, Vol. **41,** No. 1, pp. 51–58 (2023)
Number of article: 664
In print: ISSN 1857–5293
Received: September 27, 2022
On line: ISSN 1857–9191
Accepted: January 12, 2023
UDC: 004.738.5:656.13.052.43]:519.876.5
https://doi.org/10.55302/MESJ23411664051j

Original scientific paper

COMPARISON OF DIFFERENT MODELLING APPROACHES FOR VEHICLE VELOCITY CONTROL ON AN UPCOMING BOTTLENECK SECTION

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A b s t r a c t: With the development of the Internet of Things and the Smart infrastructure, simulating the traffic is not purely a mathematical question anymore, but also entails the vehicle dynamics as an important factor, due to the constant data sharing between the vehicles and the highly automated systems which control the behaviour of the vehicles. This paper aims to upgrade the traditional traffic flow model based on the Cellular Automata theory, by incorporating the positions and velocities of the vehicles, as well as introduce a longitudinal vehicle dynamics model, in order to determine the accuracy of the pure traffic model in a simulation environment (Python).

Key words: Internet of vehicles; traffic model; longitudinal dynamics; bottleneck merge

СПОРЕДБА НА РАЗЛИЧНИ НАЧИНИ НА МОДЕЛИРАЊЕ УПРАВУВАЊЕ СО БРЗИНАТА НА ВОЗИЛО ПРИ СПОЈУВАЊЕ НА ДВЕ СООБРАЌАЈНИ ЛЕНТИ ВО ЕДНА

А п с т р а к т: Со развојот на интернетот на нештата и паметната инфраструктура, симулирањето на сообраќајот повеќе не е само математички проблем, туку ја повлекува и динамиката на возилата како битен фактор, поради постојаната размена на податоци помеѓу возилата и напредните автоматизирани системи кои управуваат со однесувањето на возилата. Целта на овој труд е традиционалниот модел на сообраќаен тек базиран на теоријата на мобилни автомати, да се надгради со вклучување на позицијата и брзината на возилото вклучувајќи го и неговиот надолжен динамички модел, со цел одредување точност на чист сообраќаен модел во симулирана околина (Python).

Клучни зборови: интернет на возилата; сообраќаен модел; надолжна динамика; спојување на две ленти во една

NOMENCLATURE

IoV	Internet of Vehicles
CA	Cellular Automata
x_{new}	calculated vehicle position in the next time step
$x_{current}$	current vehicle position
Δx	calculated vehicle displacement in the next time step
Δt	time step
v_{current}	current velocity of the observed vehicle
T_f, T_r	input torque on front and rear wheel
r_d	effective tire radius
F_r	rolling resistance
m	mass of the vehicle
F_{nf}, F_{nr}	normal force on front and rear axle
l	wheelbase
V_X	vehicle longitudinal velocity
F_{xf}, F_{xr}	longitudinal tire force of front and rear axle
$I_{\omega f}$, $I_{\omega r}$	moment of inertia of front and rear wheel
ω_f, ω_r	front and rear wheel speed
F_{air}	air resistance
μ	longitudinal friction coefficient
l_f, l_r	distance from center of gravity to front and rear axles

1. INTRODUCTION

With the development of the Internet of Things and the Smart infrastructure, the idea of interconnected vehicles in the form of IoV is slowly becoming a reality. This entails the need of new models for traffic flow simulation, as well as models describing the behaviour of various automated vehicle systems that depend on the surrounding road traffic participants.

Regardess whether the observed subjects are autonomous or human-driven vehicles, one of the most common problems in the traffic flow nowadays is the traffic bottleneck situation. Merging itself is a pretty complex mathematical problem, with many possible modelling approaches, as discussed in [1]. Many authors [2, 3] suggest using the Rule 184 fuzzy cellular automation as a mathematical model of traffic flow [4]. However, this model does not take the vehicle dynamics into account, but observes the vehicles as moving points. Due to the fact that the CA modelling approach has been proven to be sufficient for traffic flow simulations, there are some authors suggesting the vehicle velocities and accelerations accompany the points in the model, using basic vehicle dynamics equations [5].

The abovementioned models could be taken one step further, by assuming that the observed vehicles are interconnected and share data via the IoV. Thus they would all be able to adjust their respective velocities in order to achieve smoother merging into a bottleneck section, as discussed in [6].

Analyzing the suggested modelling methods, the question arises whether the dynamics of the observed vehicle should necessarily be considered when simulating a bottleneck section of interconnected vehicles. The possibility exists that it is sufficient to just apply the CA model with a custom ruleset and have a discrete velocity change with a bigger time-step, i.e. that the model could capture the dynamical capabilities of a real vehicle on the road. Throughout this paper, the traffic model with the custom ruleset as well as the longitudinal vehicle dynamics model, will be presented. Furthermore, the way those two models are combined will be explained. Lastly, the simulation results will be discussed.

2. MATHEMATICAL MODELS

The main model consists of two incorporated models, one describing the behaviour of vehicles when approaching a bottleneck section, and the other describing the longitudinal vehicle dynamics. The programming language Python was used for this purpose, with the utilization of the pandas, Matplotlib and NumPy libraries.

a) *Single lane traffic model*

As mentioned earlier, the idea behind the CA model will be adopted for describing the traffic flow. However, without any modification this approach is limited, out of the scope of this research, due to the fact that it can only describe the behaviour of the vehicles in one lane. In this case, two lanes need to be observed simultaneously, since the velocity and position of the vehicles is crucial for determining the expected behaviour of the vehicles behind them.

First, a new ruleset for vehicles behaviour must be developed. It is necessary that the used algorithm takes the positions and velocities into account, meaning that a plain CA ruleset implemented on an array consisting of 0’s and 1’s will not suffice.

The first step in creating the traffic model would be getting a section of certain length (length of 300 meters is adopted throughout this paper), divided in equally long sections (3 meters each). An average length of a vehicle is considered 3 meters, due to the variety of vehicles with different geometric characteristics that can be found on the road. It is assumed that each vehicle can only take up one cell at each given moment.

The next step is populating the road, using a random function returning only 0’s (empty slot) or 1’s (populated slot), with a probability of 25%. Afterwards, looping through the slots of the lane takes place, assigning a random velocity in a certain range for every slot that contains a vehicle, as shown in Figure 1. The results are saved in a list of dictionaries, each containing two key-value pairs: one regarding the randomly assigned velocity, and the other one regarding the position (the index of the slot where the vehicle is placed).

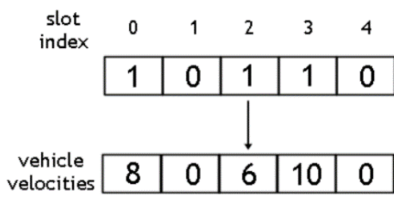

Fig. 1. Steps of populating a lane

For the purpose of the simulation, a time-step of 0.5 seconds has been adopted. After each time-step the new position of the vehicle should be calculated as a function of the velocity, using the following expression:

$$x_{new} = x_{current} + \Delta x, \quad (1)$$

where Δx is the number of the slots the vehicle passes in the given time and is calculated according to the following expression, rounded to the closest integer:

$$\Delta x = \frac{\Delta t \cdot v_{current}}{3}. \quad (2)$$

The same logic can be applied when introducing the second lane, as shown in Figure 2.

	0	1	2	3	4
lane 1	8	0	6	10	0
lane 2	0	14	0	5	9

Fig. 2. Introducing a second lane to the model

However, the actual displacement of the vehicles depends on outside factors as well, such as the velocity of the vehicle in front of it, whether or not the vehicle is ready to merge or is still not in the last slot, and the situation in the neighbour lane. The algorithm for determining the new position of the vehicle is shown in Figure 3.

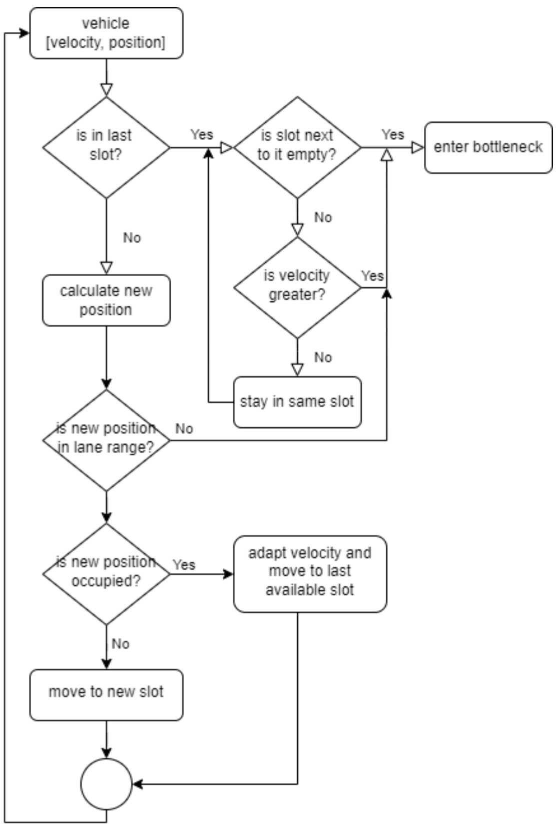

Fig. 3. Traffic flow simulation algorithm

It is important to note that when introducing the second lane, the slot with index 0 from the first lane is not randomly assigned as either 1 or 0, but is always occupied by the observed vehicle, whose dynamics should be simulated with the help of the dynamics model. Furthermore, there is a function adjusting the vehicle velocity to the one in front of it, in case of slower movement of the vehicles in the next slots of the lane and the inability to overtake, as shown in the algorithm.

If the two lanes are not observed separately, a separate function looking for the next vehicle in both lanes is to be called, and the adjusting velocity function should be called afterwards in order to slow down the vehicle, so that is could match the velocity of the vehicle which should merge before it. Doing this additional step will provide a smoother merging and traffic flow.

b) *Longitudinal vehicle dynamics model*

For the purpose of this simulation, a section with two straight lanes is observed. The vehicles in both lanes do not have the ability to overtake, nor to switch lanes, and the only time the direction of movement changes is exactly at the time of merging, which is not of interest in the simulation and is therefore neglected in the models and results. For that reason, a longitudinal dynamics model is sufficient for describing the behaviour of the vehicles in the given situation.

The longitudinal model can be applied to describe the dynamics of braking and accelerating, along with the grade angles, not considering the lateral dynamics. Moreover, the left and right wheels of the vehicle can be combined into one wheel, due to the insignificant difference between their respective wheel speeds.

The used longitudinal dynamics model can be presented with the following equations [7]:

$$\begin{cases} m \cdot \dot{v}_x = F_{xf} + F_{xr} - F_r - F_{air} \\ I_{\omega f} \cdot \omega_f = \pm T_f - r_d \cdot F_{xf} \\ I_{\omega r} \cdot \omega_r = \pm T_r - r_d \cdot F_{xr} \end{cases} \quad (3)$$

The script of the model consists of three separate classes:

1. Vehicle class, which includes the necessary vehicle parameters, such as the wheelbase, location of the centre of gravity with respect to the front and rear axles and its height, the total mass, as well as its distribution among the axes and the parameters needed to calculate the air resistance force.

2. Tire class, consisting of the necessary tire parameters, as well as functions for calculating tire slip and tire speed.

3. A Simulation class, which gets the time-step, input torque and the needed vehicle and tire parameters, in order to calculate the components describing the longitudinal behaviour of the vehicle.

The initial conditions of the vehicle dynamics simulation are shown in Table 1.

Table 1

Vehicle simulation initial conditions

Symbol	Physical quantity	Initial value	Unit
x_0	Initial position	0	kg
$\dot{x}_0$	Initial velocity	inherited from traffic model	m/s
$\ddot{x}_0$	Initial acceleration	0	m/s^2
t_0	Initial time	0	s
F_{nf}	Front axle normal force	$\frac{9.81 \cdot m \cdot l_f}{l}$	N
F_{nr}	Rear axle normal force	$\frac{9.81 \cdot m \cdot l_r}{l}$	N
F_{xf}, F_{xr}	Longitudinal forces	0	N
s_f, s_r	longitudinal slip on front and rear wheel	0	–
ω_f, ω_r	Front and rear wheel speed	$\frac{\dot{v}_{xi}}{r_d}$	rad/s
T_f, T_r	Front and rear wheel torque	0	Nm

As shown in (3), the longitudinal force, which appears as a result of the input torque, needs to be calculated in order to determine the longitudinal acceleration of the vehicle. That can be achieved using the following equation:

$$F_{xi} = F_{ni} \cdot \mu. \tag{4}$$

As can be noticed from the equation (4), the longitudinal force directly depends on two factors: the normal force, which has to constantly be re-evaluated as a function of the longitudinal acceleration, and the longitudinal friction coefficient, which depends on the longitudinal slip, i.e. the relative difference between the vehicle and the tire longitudinal velocities, and cannot be expressed empirically. There are many models describing the longitudinal behaviour of a tire, based on experimental data. In this model, the Burckhardt method is used. Coefficients used in the tire model are shown in Table 2, and the longitudinal slip-friction coefficient relation is represented by the curve shown on Figure 4.

Table 2

Coefficients used in the tire model

Symbol	Physical quantity	Value	Unit
r_d	Effective tire radius	0.3	m
φ	Wheel adhesion coefficient	0.8	–
f_0	Rolling resistance coefficient at v=0	0.015	–
I_f, I_r	Wheel moment of inertia	0.8	kgm^2
a	Burckhardt coefficients for dry asphalt [7]	1.28	–
b		23.99	–
c		0.52	–

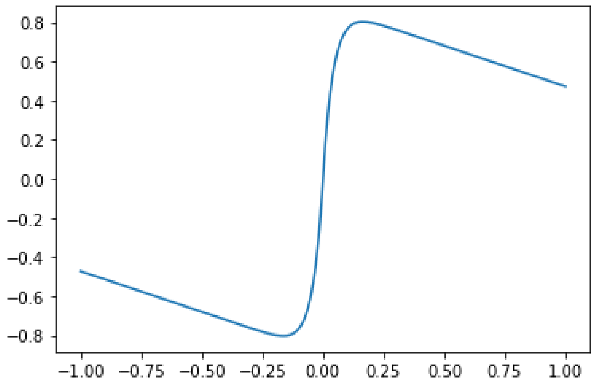

Fig. 4. Longitudinal friction coefficient in function of the wheel slip, according to the Burckhardt model (horizontal axis – wheel slip, vertical axis – friction coefficient)

c) *Incorporated model*

In order to be able to run the final simulations and get the necessary results, the two models described above need to be combined into one model, which runs according to the algorithm shown in Figure 5.

As demonstrated in the algorithm, the adopted time steps for this simulation are 0.01 seconds for the vehicle dynamics model, and 0.5 seconds for the traffic model. It is assumed that the controller gets the needed values from the surrounding vehicles twice per second, and the vehicle acts accordingly. The first check of whether the controlled vehicle is still in the lane is done by checking if the values of all the cells in lane 1, which entails our vehicle, are null. In a positive case, the observed vehicle has already gone into the merge and the simulation can be stopped, since the already presented traffic model assumes there are no vehicles behind it, i.e. that is the last vehicle of our interest in the lane.

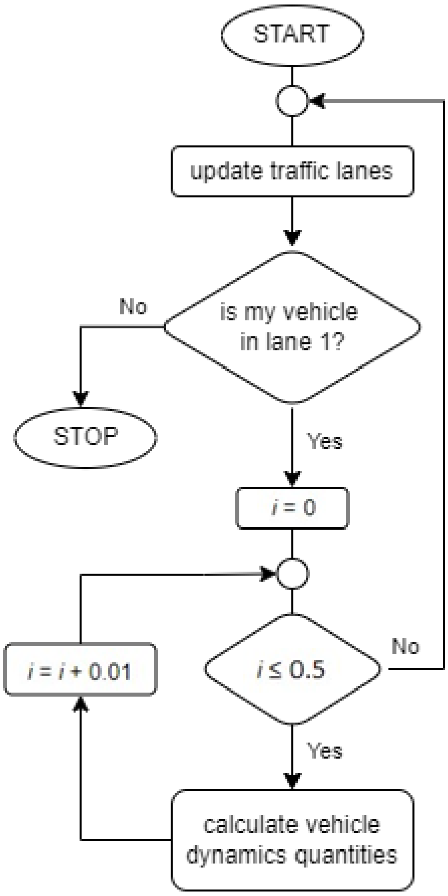

Fig. 5. Incorporated model algorithm

The outputs after every time step are stored in two separate pandas data frames, one regarding the traffic model results and the other regarding the vehicle dynamics quantities of interest, after each simulation time step.

The torque in the separate vehicle dynamics model needed to be hardcoded for each simulation, in order to imitate the behaviour of the driver. However, due to the random population of the lanes, it is impossible to know the traffic situation, and thus the needed torque, beforehand. That is why a logic for determining whether or not the torque should be increased, i.e. decreased, in the next time step and by how much, needs to be implemented. It is important to note that, in order to simplify the model, it is assumed that the vehicle has a non-zero input torque at any given time during the simulation. The velocity corrections will be done exclusively by adding a positive or negative torque, which will possibly result in unnecessary velocity dilatations that would not provide a comfortable ride, but are sufficient for the purpose of this research.

The target torque for the next time step can be calculated right after reading the velocity and position values of both the controlled vehicle and the one in front of it. Regardless of whether it is located in the same lane or not. The 2-second rule is followed in this model, meaning that if the distance between the two vehicles is greater than 2 seconds at the current velocities, the target torque should be 2000 Nm, which is achievable by most vehicles and still provides a smooth acceleration, thus allowing the vehicle to close the gap until it reaches a 2-second distance.

In case the first requirement is met, the next step should be adapting the velocity of the controlled vehicle with regards to the data gathered about the vehicle in front of it. Namely, the needed acceleration or deceleration can be calculated by simply implementing the following equation:

$$a = \frac{v_{target} - v_{current}}{\Delta t}, \tag{5}$$

where a is the target acceleration, v_{target} is the target velocity (the velocity of the vehicle in the front), $v_{current}$ is the current velocity of the controlled vehicle and Δt is the traffic model time step. Knowing the acceleration value, the inertial force F_i can be easily calculated. Furthermore, the air and rolling resistance forces can be calculated at target velocity.

Given the nature of the longitudinal dynamic vehicle model, the needed torque can be calculated by balancing the forces along the longitudinal axes, thus getting the following equation:

$$T_{target} = \pm(F_i + F_r + F_{air}) \cdot r_d. \tag{6}$$

Once the target torque is known, in order to achieve a smoother transition, provided we know the input torque from the last time step of the previous simulation cycle, T_{prev}, a slightly modified version of the Sigmoid function [8, 9] can be used:

$$sig = \frac{T_{target} - T_{prev}}{1 + e^{-0.02t+6}} + T_{prev}. \tag{7}$$

Given that the majority of the cars are front-wheel drive, but break using all four wheels, it is safe to adopt the same logic in this model. However, this entails the need to incorporate a brake torque distribution model between the front and rear axles. First of all, it needs to be determined whether the torque is increasing or decreasing, i.e. whether the vehicle is accelerating or braking. This can be done with the following check:

$$\begin{cases} T_{target} < T_{prev} => braking \\ T_{target} > T_{prev} => accelerating \end{cases} \tag{8}$$

If it is determined that the vehicle will be accelerating in the following time step, the needed torque as a whole should be applied to the front axle, whereas the rear axle gets a torque of 0. On the other hand, if the brakes need to be applied, the distribution among the axles should follow the following rule:

$$\begin{cases} T_{front} = \frac{T_{sig} \cdot F_{nf}}{9.81 \cdot m} \\ T_{rear} = T_{sig} - T_{front} \end{cases}, \quad (9)$$

where T_{sig} signifies the input torque from the Sigmoid function at the current time step.

3. SIMULATION

In this section, the simulation results will be presented and discussed.

a) *Analysis of the vehicle dynamics model output in comparison to the gathered IoV data*

As mentioned earlier, one of the goals of this simulation is to check if the vehicle on the road can follow along with other vehicles presented as simple points, in terms of dynamics. That is why two simulations with different velocity ranges have been run and only the velocity comparison between the controlled vehicle and the one in front of it will be presented, as it is the only quantity of interest.

The first simulation entails vehicles distributed among two traffic lanes, as explained in-depth in the previous section, with velocities in the range between 14 and 28 m/s, to simulate traffic flow on a motorway (Figure 6), whereas velocity range in the second simulation is between 4 and 14 m/s, to simulate city traffic (Figure 7).

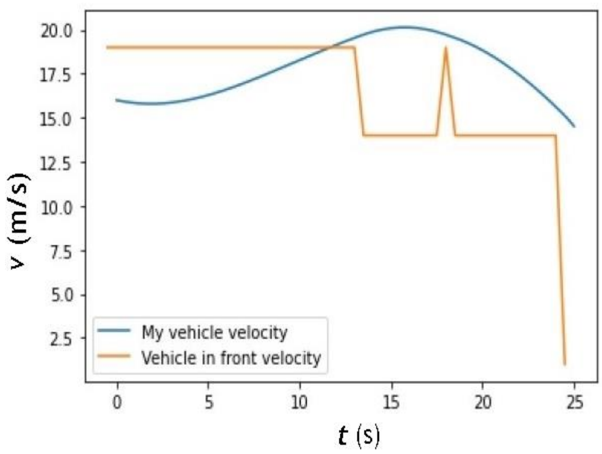

Fig. 6. Velocity comparison on a motorway

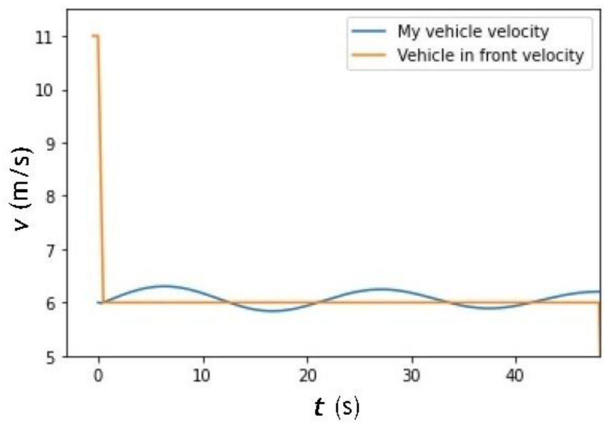

Fig. 7. Vehicle comparison on city road traffic

The same can be said about the second simulation, whose results are shown in Figure 7. Nonetheless, it is important to note that, due to the assumption that the vehicle has an input torque at any given time, as mentioned in the previous section, slight dilatations in the vehicle velocity can be noticed. One of the main reasons is the fact that the traffic situation in this instance is such that the vehicle in the front has a constant speed throughout the whole simulation, which is almost the same as the starting velocity of the controlled vehicle.

b) *Comparison between the vehicle dynamics model outputs and the behaviour of the same vehicle in the traffic model*

It is also of great significance to check whether is it really essential to incorporate the longitudinal dynamics model, given that it significantly burdens the simulation as a whole and is thus a far more expensive process. It can be determined whether the dynamics model could be omitted without disturbing the validity of the simulations, by simply comparing the results of both models (one considering the vehicle dynamics, and the other one only containing the traffic model, regardless of the dynamical capabilities of the vehicle) and checking if they are similar enough to suffice for the given purpose.

In order for this conclusion to be drawn definitely, the comparison needs to be done on a setup with more variable requirements, i.e. the velocity of the vehicle in the front should be changing frequently and thus, the response of the system can be evaluated more accurately. For this reason, a simulation where the velocity of the vehicle in the front is decreasing over time, has been chosen.

As can be noticed from the results presented on Figure 8, the controlled vehicle, due to the nature of the traffic model, immediately adapts the velocity according to the one of the vehicle that should merge before it, and follows that velocity throughout the whole simulation. On the other hand, the model on Figure 9 presents a slightly different behaviour. Namely, the velocity of the vehicle in the front is also followed throughout the whole simulation, but that process does not happen immediately, due to the dynamical characteristics of the vehicle.

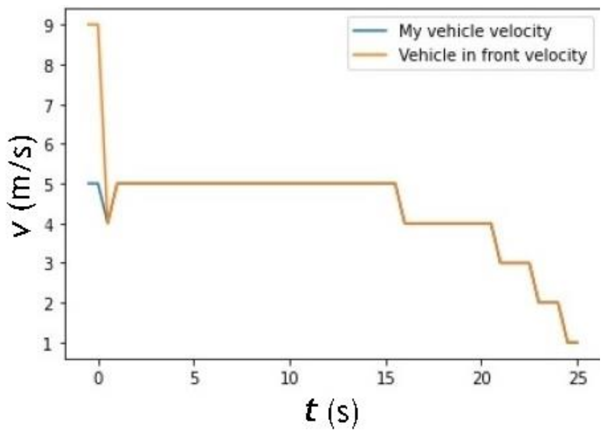

Fig. 8. Comparison between the velocities of the controlled vehicle and the one in the front, drawn from the model without the longitudinal dynamics logic

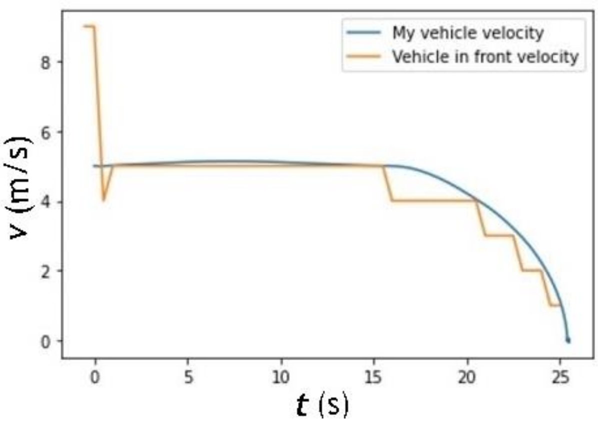

Fig. 9. Comparison between the velocities of the controlled vehicle and the one in the front, drawn from the model including the longitudinal dynamics logic

If both results are compared, such as in Figure 10, the conclusion can be drawn that, even though the first results are not as dynamically accurate and do not reflect the real behaviour of the vehicles as the second one, both curves still have clashing points throughout the whole simulation, and only differ in terms of the paths leading to those points. This leads to the conclusion that, if one is only interested in the behaviour of the vehicles in this particular traffic situation, and how they interact with each other if interconnected via the *IoV*, then the pure traffic model would suffice and does not need to be burdened with the unnecessary dynamics model. Nevertheless, if the exact behaviour of the vehicle is of significance, the results show that this way of modelling and running the simulations is necessary.

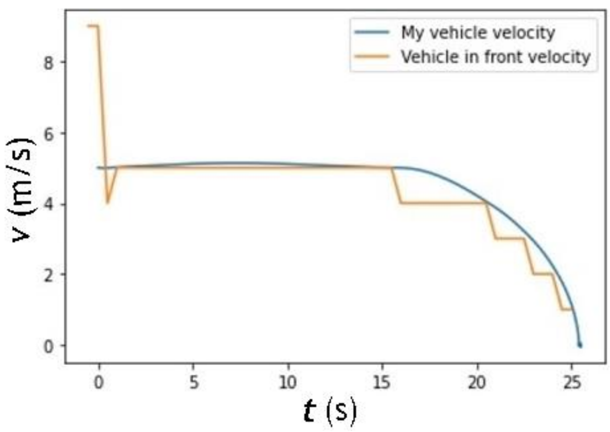

Fig. 10. Comparison between the velocities of the controlled vehicle, drawn from both models

4. SUMMARY AND CONCLUSIONS

The modelling and simulation of the control and behaviour of a vehicle, interconnected with the other vehicles on the road via the *IoV*, on a traffic section with an upcoming bottleneck merge, was shown in this paper. The modelling included both a mathematical description of a traffic flow behaviour, on the basis of the CA method, but with a new ruleset, as well as a longitudinal dynamics model for describing the behaviour of the controlled vehicle in particular.

There were mainly two points of focus, the first one being whether the vehicle is able to follow the changes introduced discretely by the traffic model, in terms of vehicle dynamics, and the second one whether this step is crucial or can it be omitted, without affecting the accuracy of the simulations.

Given the results in the last section, it can be concluded that, depending on the needs of the simulation, a pure traffic model based on the CA theory, but with a ruleset incorporating the positions and velocities of all vehicles in all given times, as well as for updating the lane cells appropriately, could be sufficient. Nevertheless, for vehicle control purposes, where many physical quantities are crucial and need to be knows at all times, the longitudinal dynamics model can be implemented, without introducing disturbances in the traffic model.

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