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Original scientific paper

CONTRIBUTION ON INVESTIGATING THE POSSIBILITY OF USING TWO-PARAMETER FREQUENCY ANALYSIS IN EXPERIMENTAL PARAMETER IDENTIFICATION OF TORQUE VIBRATIONS OF VEHICLE CARDAN SHAFTS

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A b s t r a c t. During exploitation, motor vehicles are subjected to vibration loads that lead to fatigue of their users and materials of their aggregates. Therefore, vibrations must be studied from the earliest stage of development, using mathematical models, experiments, or their combinations. In theoretical studies, vibrations of concentrated masses are usually observed, although, with the development of numerical methods (especially finite element method), attention is paid to vibrations of elastic vehicle systems. Then, idealizations are usually made, especially regarding operational conditions and relationships between motor vehicle aggregates. In this paper, an attempt was made to develop a method for identifying real vibration loads of elastic vehicle cardan shafts under operational conditions. Namely, 2D Fourier transformation was used for two-parameter frequency analysis. The possibility of the procedure's application was demonstrated on an idealized elastic cardan shaft. The research showed that two-parameter frequency analysis can be used to generate torsional vibrations of elastic vehicle cardan shafts in laboratory conditions.

Key words: vehicle; elastic cardan shaft; torsional vibrations; two-parameter frequency analysis

ПРИЛОГ КОН ИСТРАЖУВАЊЕ НА МОЖНОСТА ЗА КОРИСТЕЊЕ АНАЛИЗА НА ФРЕКВЕНЦИЈАТА СО ДВА ПАРАМЕТРА ЗА ЕКСПЕРИМЕНТАЛНА ИДЕНТИФИКАЦИЈА НА ПАРАМЕТРИТЕ НА ВРТЕЖНИОТ МОМЕНТ НА КАРДАНСКИТЕ ОСКИ НА ВОЗИЛОТО

А п с т р а к т. За време на експлоатацијата, моторните возила се подложени на вибрациони оптоварувања што доведуваат до замор на материјалите на нивните агрегати. Затоа вибрациите мора да се проучуваат уште од најраната фаза на развој, користејќи математички модели, експерименти или нивни комбинации. Во теоретските студии обично се разгледуваат вибрации на концентрирани маси, иако, со развојот на нумеричките методи (особено методот на конечни елементи), им се посветува внимание и на вибрациите на еластичните системи на возилата. Потоа обично се прават идеализации, особено во однос на условите за работа и односите меѓу агрегатите на моторните возила. Во овој труд беше направен обид да се развие метод за идентификување на реалните вибрациони оптоварувања на еластичните кардански вратила на возилото при работни услови. Имено, за анализа на фреквенцијата со два параметра се користеше 2D Фуриеова трансформација. Можноста за примена на постапката беше демонстрирана на идеализирана еластична осовина. Истражувањето покажа дека анализата на фреквенцијата со два параметра може да се користи за да се генерираат торзиони вибрации на еластичните вратила на возилото во лабораториски услови.

Клучни зборови: возило; еластично вратило; торзиони вибрации; анализа на фреквенција со два параметра

1. INTRODUCTION

During exploitation, motor vehicles are subjected to vibration loads that lead to fatigue of their users and materials of their aggregates. Therefore, vibrations must be studied from the earliest stage of development, using mathematical models, experiments, or their combinations.

In theoretical studies, vibrations of concentrated masses are usually observed, although, with the development of numerical methods (especially finite element method), attention is paid to vibrations of elastic vehicle systems. Then, idealizations are usually made, especially regarding operational conditions and relationships between vehicle aggregates [1].

The specificity of vehicle operational conditions is their random character [1], which significantly complicates theoretical considerations using models, so experiments are practical and irreplaceable. Namely, despite significant progress in developing software for automatic vehicle design and calculation [4], the final judgment on their characteristics is based on experimental research. Therefore, experimental methods are still significant today.

When it comes to elastic vehicle cardan shafts subjected to torsional vibrations, a problem often arises in identifying the parameters of these vibrations. Methods for identifying them are developed, as is the case with modal analysis [5–10]. In practical terms, vibration modes are determined in laboratory conditions. However, a problem arises in the case when actual exploitation conditions are necessary to generate the torsional loads of the cardan shaft on test benches, as the modal analysis does not provide sufficient opportunities for generating these signals in the time domain.

Therefore, it was deemed useful to develop a procedure for identifying the parameters of torsional vibrations of elastic vehicle cardan shafts, which would enable their generation in laboratory conditions.

One possibility is frequency analysis using the Fourier transform, which enables the determination of the frequency content of signals by calculating the spectra magnitudes and phase angles [11, 12], that allow the generation of an original, time-dependent signal using the inverse Fourier transform, which is routinely performed in cases where the signal depends only on time [11].

However, vibrations of elastic systems depend on multiple parameters (dimensions and time), suggesting that a multi-parameter Fourier transform must be used. In the case of an idealized cardan shaft (with other types of vibration ignored), torsional vibrations change along the length of the shaft and depend on time, so the so-called two-parameter Fourier transformation (2D) must be applied [13, 14].

This paper will analyze the possibility of using a two-parameter Fourier transform to create conditions for studying vibrations of elastic vehicle cardan shafts in laboratory conditions.

Therefore, a general expression for the Fourier transform in case of multiple variables will be given [15]:

$$F(\xi_1,\xi_2 \dots . \xi_n) = \int_{R^n} e - 2\pi i(x_1\zeta_1 + x_2\zeta_2 + $$
$$+ \cdots x_n\zeta_n) * f(x_1, x_2 \dots . x_n) dx_1 dx_2 **** dx_n \quad (1)$$

where:

$f(x_1, x_2 \dots . x_n)$ – a function of n variables,

$x_1, x_2, \dots . x_n$ – variables,

$\xi_1, \xi_2 \dots \dots \xi_n$ – circular frequency, and

$\int_{R^n}$ – multiple integrals (double for 2D, triple for 3D, etc.).

2. METHOD

As previously mentioned, this paper aims to investigate the possibility of using two-parameter frequency analysis (2D Fourier transformation) in identifying parameters of torsional vibrations of elastic vehicle cardan shafts. In the absence of experimental data on registered torsional vibrations of the shaft, the method is illustrated with data obtained from a dynamic simulation using its mathematical model. As is known, vibrations of elastic elements are described by partial differential equations [13,14]. For further consideration, Figure 1 will be observed.

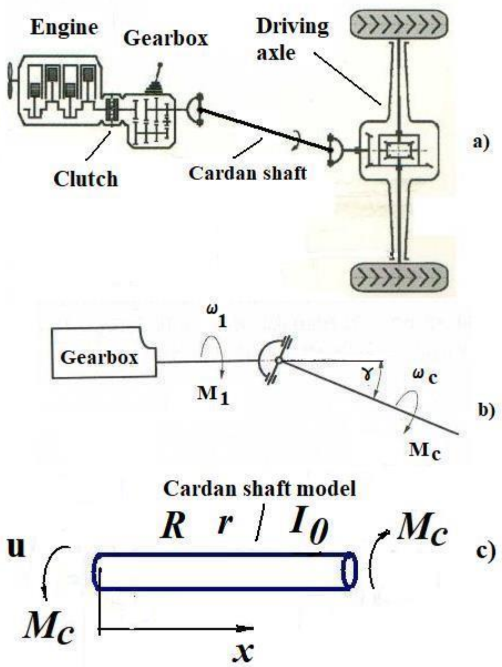

Fig. 1. The concept of transmission (1 a), partial scheme of cardan shaft (1 b), and cardan shaft model (1c)

Figure 1 shows the concept of transmission of the observed commercial motor vehicle (1a). Given that this transmission concept is widely applied in practice, it has been deemed appropriate not to give any further explanations. The cardan shaft is intended to transmit the torque from the gearbox to the drive axle in cases where their axes do not overlap, as illustrated in Figure (1b). A simplified model of an elastic cardan shaft is shown in Figure (1c).

When defining the model to describe the torsional vibrations of the elastic cardan shaft, the following assumptions were made:

– the influence of its mass on the occurrence of transverse vibrations was neglected,

– the shaft was cylindrical (tube) with a constant outer and inner diameter along its length,

– the shaft was completely dynamically balanced and the influence of clearance in the joints was neglected, but friction losses in the joints of the shaft were included.

Given that the partial differential equations that describe torsion vibrations of elastic bodies, which also applies to the cardan shaft, is described in detail in [13, 14], it will not be done here, but its final form will be given. Given the assumptions made, forced torsional vibrations of the elastic cardan shaft [13,14] are described by the partial differential equation:

$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2} + f(x,t), \tag{2}$$

where:

$u\,(x, t)$ – torsional vibrations of the cardan shaft,

x – coordinate along the length of the shaft,

$f(x, t)$ – forced torque originating from unbalanced motor forces and the random character of micro-roughness,

t – time, and

$$c^2 = \frac{G}{\rho},$$

where:

G – shear modulus, and

ρ – density of the shaft material.

As is known [13,14,16], to find the general integral of the partial differential equation (2), it is necessary to know the boundary and initial conditions. As is known, the torsional torque caused by vibrations of the cardan shaft can be expressed [13, 14, 16]:

$$M = GI_0 \frac{\partial u(x,t)}{\partial x}. \tag{3}$$

where:

I_0 – a polar moment of inertia given by the expression for a circular ring cross-section:

$$I_0 = \frac{\pi(R^4 - r^4)}{2}$$

where:

R – outer, and

r – inner radius of the cardan shaft tube.

The left boundary condition of the cardan shaft is defined by the equality of the output torque from the gearbox and the torsional torque transferred to it. The right boundary condition of the cardan shaft is defined by the equality of the torque that needs to be brought to the drive axle and the torsional torque of the shaft.

For further consideration, Figure 1b will be observed.

Without delving into the theory of spherical motion of the universal joint, which is extensively explained in [17, 18], the vector of the angular velocity of the output shaft of the gearbox is projected onto the axis of the cardan shaft, and the following relationship applies:

$$\omega_c = \omega_1 \cos(\gamma)$$

where γ is the angle of the universal joint.

The opposite situation occurs at the right end, where the following relationship can be written for the angular velocity of the input shaft of the drive shaft:

$$\omega_c = \omega_2 \cos(\gamma),$$

where ω_2 the angular velocity of the drive shaft.

To define the boundary conditions, it is necessary to calculate the output torque from the gearbox and the input torque to the drive axle. Based on the power equality that is transmitted from the gearbox to the left cross joint of the cardan shaft, we have:

$$M_1\omega_1 = M_c\omega_c = M_c\omega_1 \cos(\gamma). \tag{4}$$

The same can be written for the right cross joint of the cardan shaft (where the influence of friction is included in the joints of the cardan shaft via the efficiency factor η_c):

$$M_c\omega_c\eta_c = M_2\omega_2 = M_2 \frac{\omega_c}{\cos(\gamma)}. \tag{5}$$

where M_2 is the input torque to the drive axle.

The torque M_2 will be calculated from the traction balance during slow uniform motion of the observed commercial motor vehicle on a road with a longitudinal slope defined by the longitudinal angle α. Given the conditions of the observed vehicle motion, the required moment on the input shaft of the drive axle M_2 is defined by the expression [19]:

$$M_2 = \frac{mg(f\cos\alpha + \sin\alpha)r_d}{i_0\eta_0}. \quad (6)$$

where:

m – the mass of the vehicle,

g – the acceleration due to gravity,

α – the longitudinal slope angle,

f – the coefficient of rolling resistance,

i_0 – the gear reduction in the drive shaft,

η_0 – the efficiency of the drive axle.

Based on expressions (3), (4), and (6), the boundary condition for the left end of the cardan shaft is obtained:

$$\frac{\partial u(x,t)}{\partial x} = \frac{1}{GI_0}\frac{M_1}{\cos(\gamma)}, \quad (7)$$

while based on expressions (3), (5), and (6), the boundary condition for the right end of the cardan shaft can be written:

$$\frac{\partial u(x,t)}{\partial x} = \frac{1}{GI_0}\frac{M_2}{\eta_c\cos(\gamma)}. \quad (8)$$

For the left end of the shaft, $x = 0$ should be placed, and for the right end, $x = L$ (where L is the length of the cardan shaft).

The following initial conditions were assumed for the dynamic simulation:

$$u(x,t) = 0; \quad \frac{\partial u(x,t)}{\partial t} = 0 \quad (9)$$

for $t = 0$.

It was deemed appropriate to use a forced torque (excitation function) in partial differential equation (2) that takes into account the imbalance of the engine's torque or the random nature of the longitudinal micro-roughness of the road.

More precisely, in the absence of real data, it was assumed that the engine torque changes with twice the frequency of the number of revolutions (the so-called second harmonic), and that the effect of longitudinal road roughness can be represented by a random function [19], i.e.:

$$f(x,t) = a_m\sin(4\pi nt)$$
$$f(x,t) = a_m[(rnd - 0.5) + \sin(4\pi nt)]$$

where:

a_m – amplitude,

rnd – random numbers uniformly distributed in the interval 0,1,

n – number of engine revolutions, and

t – time.

The partial differential equation (2), with boundary and initial conditions (7), (8), and (9), can be solved only in the case of harmonic excitation [13, 14], so an attempt was made to solve it using the Wolfram Mathematica 13.2 program [15]. However, difficulties arose with listing numerical data, so it was decided to solve the problem numerically [20], using the finite difference method. As this procedure is known from [20], it will not be discussed here, and the problem was solved using a developed program in Pascal.

The dynamic simulation was performed for a steel elastic cardan shaft, using the following data: m = 22000 kg; i_0 = 7.85; i_I = 6.87; r_d = 520 mm; η_0 = 0.90; η_c = 1; G = 8·104 N/mm^2; ρ = 8·10–6 kg/mm^3; R = 125 mm, r = 100 mm; n_x = 256; h_x = 5 mm; n_t = 256; h_t = 0.01 s; a_m = 20 Nm; ρ = 6 °.

As torsional vibrations of the elastic cardan shaft depend on two parameters, 3D graphics are required to represent them graphically. For illustration, the results of the numerical integration of the partial differential equation (2) are shown for the used boundary and initial conditions in Figures 2 and 3.

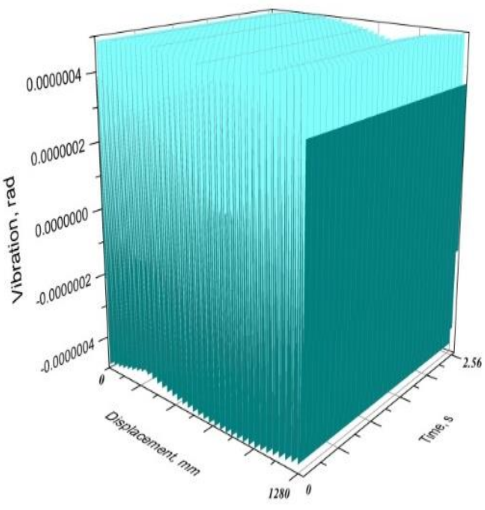

Fig. 2. Torsional vibrations of the cardan shaft for the forced torque $f(x, t) = a_m$·sin($2nt$)

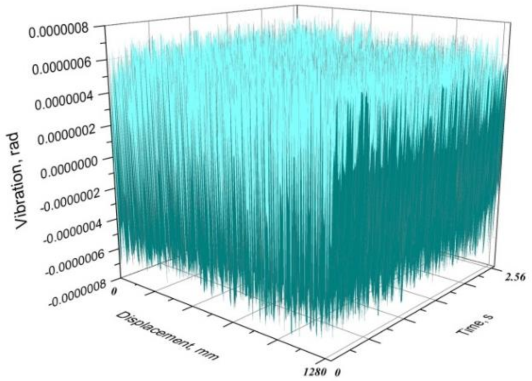

Fig. 3. Torsional vibrations of the cardan shaft for the forced torque $f(x,t) = a_m[\sin(2nt) + (rnd–0.5)]$

In Figure 2, harmonic waves along the length of the shaft can be observed due to the unbalanced second harmonic of the motor, which is following the theoretical solutions from [13, 14].

Figure 3 shows the simultaneous effect of the unbalanced engine torque and road microroughness on the torsional vibrations of the elastic cardan shaft, but in this case, randomly-shaped waves appear.

Since the torsional vibrations of the elastic cardan shaft depend on two parameters (displacement x and time t), it is necessary to apply 2D Fourier transformation. To implement it, the author developed software in Pascal. However, considering the available commercial software on the market, it was deemed appropriate to use Origin 8.5 [21] in further analyses, as potential users will have easier access to that software.

Using the mentioned software, the spectra magnitudes and phases of the two-parameter Fourier transformation were calculated, and, for illustration purposes, the results are shown in Figures 4–7.

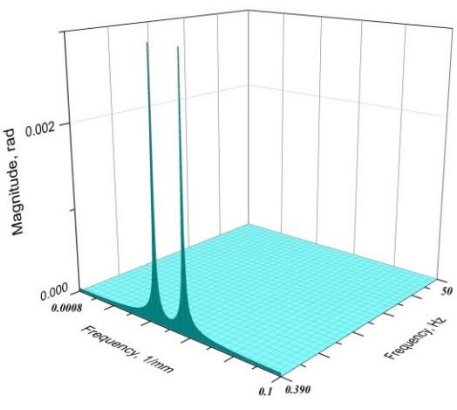

Fig. 4. The module spectrum of torsional vibrations of the cardan shaft for the forced torque $f(x,t) = a_m*\sin(2nt)$

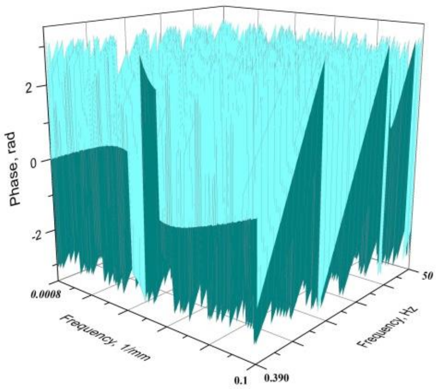

Fig. 5. The phase angle of torsional vibrations of the cardan shaft for the forced torque $f(x,t) = am*\sin(2nt)$

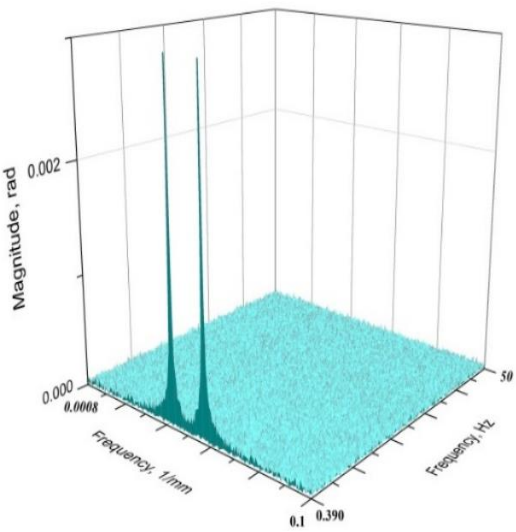

Fig. 6. The module spectrum of torsional vibrations of the cardan shaft for the forced torque $f(x,t) = a_m[\sin(2nt) + (rnd–0.5)]$

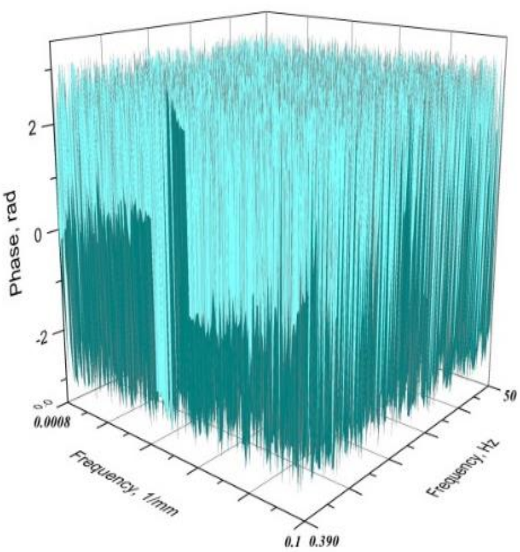

Fig. 7. The phase angle of torsional vibrations of the cardan shaft for the forced torque $f(x,t) = a_m[\sin(2nt) + (rnd–0.5)]$

3. DATA ANALYSIS

Based on the data analysis shown in images 4-7, it can be concluded that the spectra magnitude modules and phase angles describe the wave character of torsional vibrations of the elastic propeller shaft, which is consistent with the theoretical solutions from [13, 14].

The waves are more clearly visible in cases of harmonic disturbance of the form $a_m \cdot \sin(2nt)$, while in the case of using a disturbance function $a_m[\sin(2nt) + (rnd - 0.5)]$, the waves are random, as expected.

Based on previous analyses, it can be claimed that 2D Fourier transformation reliably enables data analysis of torsional vibrations of the elastic cardan shaft, which can have practical applications, as the inverse Fourier transformation enables laboratory generation of identical vibrations in operational conditions [22]. The inverse Fourier transformation can be realized using the aforementioned software Origin 8.5 [21].

During operational testing, it is necessary to register torsional vibration parameters of the elastic cardan shaft (stress, angular displacement, speed, or acceleration) along its length, over longer periods. Minimum and maximum frequency values depend on the length of the shaft, i.e. length of the time signal and discretization step.

First, it is necessary to adopt the maximum interesting frequencies fx_{max} and ftmax, then the setting step of the transducer and sampling of the time signal is defined based on the equation (Nyquist frequency) [11]:

$$h_x = \frac{1}{2fx_{max}} \qquad h_t = \frac{1}{2ft_{max}}.$$

The minimum interesting frequency is determined based on the length of the shaft ($L = nx \cdot hx$) or the length of the time signal ($T = nt \cdot ht$), according to the expressions:

$$f_{x_{min}} = \frac{1}{L} \qquad f_{t_{min}}$$

It should be noted that there are no explicit procedures for calculating spectral analysis errors for two-parameter Fourier transforms, unlike in the case of one-dimensional Fourier transforms [11]. Taking this into account, as well as the fact that this paper aims to illustrate the possibilities of applying two-parameter frequency analysis in the study of torsional vibrations of elastic cardan shafts in vehicles, analysis of statistical errors was not performed in detail.

The developed procedure has created conditions for analysis of the influence of the integration step on the accuracy and stability of partial differential equation (2) solutions, the influence of design parameters on torsional vibrations of elastic cardan shafts, the influence of forced torques, and so on. However, considering that the results of dynamic simulation in this paper served as a replacement for missing experimental results, it was evaluated that a more detailed analysis is not necessary.

4. CONCLUSION

Based on the conducted research, it can be stated that the two-parameter Fourier transform reliably enables the analysis of experimental data on torsional vibrations of elastic cardan shafts.

The calculated spectra magnitudes and phase angles, with the application of inverse 2D Fourier, transform, enable the generation of identical vibrations in the laboratory as well as in exploitation conditions.

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